Thm: A G_δ set in a compact T_2 space is Baire

Corollary: ① A complete metric space is Baire
② A locally compact T_2 space is Baire

Pf of ①: If X is completely metrizable, then X is G_δ in βX, a compact T_2 space, so X is Baire ∎

Pf of ②: If X is locally compact T_2, then X is $T_{3\frac{1}{2}}$, so X embeds into it's SC compactification βX. Let $\varphi : X \to \beta X$ be the embedding. Then $\varphi(X)$ is locally compact in βX, and in a T_2 space this implies $\varphi(X) = U \cap F$ for some open U, some closed F. Note $\overline{\varphi(X)} = \beta X$, but $\varphi(X) \subseteq F \Rightarrow \overline{F} = \beta X$, but F closed $\Rightarrow \overline{F} = F$, so $F = \beta X$, so $\varphi(X) = U \cap \beta X = U$, ie $\varphi(X)$ is open in βX. But of course open sets are G_δ, so $\varphi(X)$ is a G_δ set in a compact T_2 space, ie Baire, so X is also Baire. ∎